

Coordinate Geometry

Straight line

⇒ Every 1st degree eqⁿ like $ax + by + c = 0$ would be the equation of straight line

Slope of a line

Slope (m) of a non-vertical line passing through the pts. (x_1, y_1) and (x_2, y_2) is given by.

$$m = \frac{y_1 - y_2}{x_1 - x_2}, \text{ for } x_1 \neq x_2$$

⇒ If a line makes an angle α with the +ve direction of x axis, then the slope of line is given by

$$m = \tan \alpha, \quad \alpha \neq 90^\circ$$

Imp slope of a horizontal line is zero, and slope of vertical line is undefined.

⇒ An acute angle (say θ) between lines L_1 & L_2 with slopes m_1 and m_2 is given by

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|, \quad 1 + m_1 m_2 \neq 0$$

⇒ Two lines are Parallel if and only if their slopes are equal i.e. $\boxed{m_1 = m_2}$

⇒ Two lines are Perpendicular (⊥) if and only if product of their slopes is -1 i.e.

$$m_1 m_2 = -1$$

Imp Three points ABC are collinear, if and only if
slope of AB = slope of BC

Various forms of eqⁿ of a line

(1) Two point form :- Eqⁿ of line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)}$$

(2) Slope Intercept form :-

The point (x, y) on the line with slope m and y intercept c lies on the line if and only if $\boxed{y = mx + c}$

(3) Intercept form :- Equation of line making intercepts a and b on the x and y axis, resp. is

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

(4) Normal form :- The equation of line having normal distance from origin ' p ' and angle b/w normal and +ve x axis ' ω ' is given by $\boxed{x \cos \omega + y \sin \omega = p}$

(5) General eqⁿ of a line :- Any eqⁿ of the form $Ax + By + C = 0$, with A & $B \neq 0$ simultaneously, is called general linear equation

(6) Point of intersection of two lines :-

Let equation of lines be $ax_1 + by_1 + c_1 = 0$ & $ax_2 + by_2 + c_2 = 0$
then their point of intersection is

$$\left(\frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}, \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1} \right)$$

(7) Perpendicular Distance formula :-

If two lines of slopes m_1 and m_2 are \perp or, then the angle b/w the lines θ is of 90° therefore $\cot \theta = 0$

$$\boxed{\frac{1 + m_1 m_2}{m_2 - m_1} = 0} \Rightarrow \boxed{1 + m_1 m_2 = 0} \Rightarrow \boxed{m_1 m_2 = -1}^*$$

Assignment

1. Find the slope of a straight line which passes through points $(-5, 7)$ and $(-4, 8)$.
2. Find the slope and y -intercept of the straight line $4x - 7y + 1 = 0$
3. Find the equation of the straight line joining the pts. $(-3, 4)$ and $(5, -2)$
4. Find the equation of the line which cuts off the intercept 3 on the +ve direction of x axis and an intercept 5 on the -ve direction of y -axis.
5. Find the equation of the straight line which is at a distance of 7 units from the origin and the perpendicular from the origin to the line makes an angle of 45° with the +ve direction of x axis.
6. Find the coordinates of point of intersection of the lines $2x - y + 3 = 0$ and $x + 2y - 4 = 0$.
7. Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$.
8. Two lines passing through the point $(2, 3)$ intersect each other at an angle of 60° . If slope of one line is 2, find the equation of the other line
9. Let $P(6, 4)$ and $Q(2, 12)$ be the two points. Find the slope of a line \perp to PQ