Straight lines Contd....

Perpendicular distance (d) of a line $Ax+By+C=0$ from a pt (x_1,y_1) is given by

$$d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

(8) Equation of Bisectors of angle b/w two straight lines.

Equation of Bisectors b/w the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are given by

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm \frac{a_2+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

GENERAL EQUATION OF CIRCLE :-

circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane is constant

The general equation of a circle is given by

$x^2+y^2+2gx+2fy+c=0$, where the centre of the circle $= (-g,-f)$ and Radius of circle $= \sqrt{g^2+f^2-c}$

⇒ If $g^2+f^2-c>0$ then the radius of circle is real and hence the circle is also real.

⇒ If $g^2+f^2-c=0$, then the radius of circle is 0 and the circle is known as point circle.

⇒ If $g^2+f^2-c<0$ then the radius of the circle is imaginary and the circle is also imaginary which is not possible to draw.

Standard form of a circle :-

Eqⁿ of circle having centre (h,k) and radius 'a' is

$$(x-h)^2+(y-k)^2=a^2$$

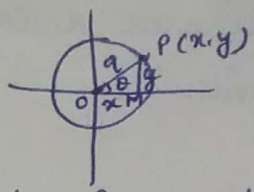
If centre is at origin $(0,0)$ then the eqⁿ becomes

$$x^2+y^2=a^2$$

Parametric equation of circle.

for centre at origin eq^n of circle is

$$x^2+y^2=a^2$$



$$\boxed{x=a\cos\theta}, \quad \boxed{y=a\sin\theta} \quad 0\le\theta\le 2\pi$$

→ This is the Parametric equation for $x^2+y^2=a^2$

⇒ Equation of circle when centre and radius is given

The eq^n $x^2+y^2+2gx+2fy+c=0$ represents a circle whose centre is $(-g,-f)$ i.e., $\left(\frac{-1}{2}\text{coeff. of } x, \frac{-1}{2}\text{coeff. of } y\right)$ and

$$\text{radius} = \sqrt{g^2+f^2-c} = \sqrt{\left(\tfrac{1}{2}\text{coeff of } x\right)^2+\left(\tfrac{1}{2}\text{coeff. of } y\right)^2-c}$$

⇒ Equation of circle passing through three given points $(x_1,y_1),(x_2,y_2)$, and (x_3,y_3) is

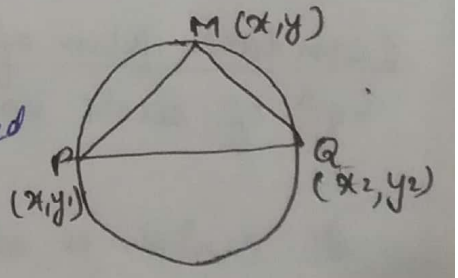
$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

⇒ Equation of circle when coordinates of end pt. of a diameter are given:

Let the two given pts be $P(x_1,y_1)$ and $Q(x_2,y_2)$

We have to find eq^n of the circle for which the line segment PQ is a diameter.

Let $M(x,y)$ be any point on the required circle. Join PM and MQ.



m_1 = slope of PM $= \frac{y-y_1}{x-x_1}$

m_2 = " " PQ $= \frac{y-y_2}{}$

Contd:-

Since the angle subtended at the point M in the semi-circle PMQ is a right angle.

Now, PQ is a diameter of the required circle.

So $\angle PMQ = 90^\circ$ i.e PM is $\perp$ ar to QM

$$\therefore \quad \frac{y-y_1}{x-x_1} \times \frac{y-y_2}{x-x_2} = -1 \qquad [m_1 m_2 = -1 \text{ when lines are } \perp \text{ ar}]$$

$$\Rightarrow (y-y_1)(y-y_2) = -(x-x_1)(x-x_2)$$

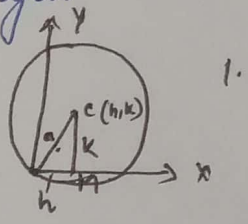
$$\Rightarrow \boxed{(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0}$$

This is the required eqⁿ of circle having (x_1, y_1) and (x_2, y_2) as the co-ordinates of the end pts of a diameter.

Some other standard forms :-

1. Equation of circle which passes through the origin

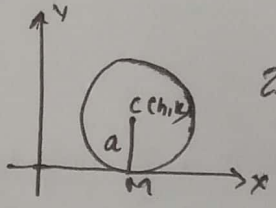
$$x^2 + y^2 - 2hx - 2ky = 0$$



1.

2. Equation of circle which touches the x-axis

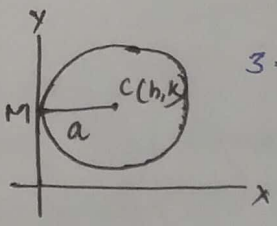
$$x^2 + y^2 - 2hx - 2ay + h^2 = 0$$



2.

3. Equation of circle which ~~toy~~ touch Y axis

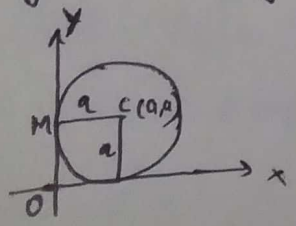
$$x^2 + y^2 - 2ax - 2ky + k^2 = 0$$



3.

4. Equation of circle which touches both the axis

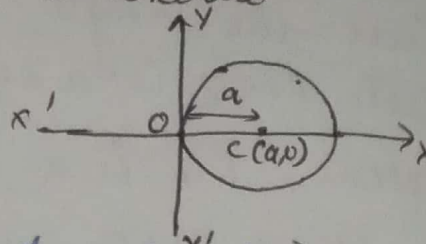
$$x^2 + y^2 - 2ax - 2ay + a^2 = 0$$



4.

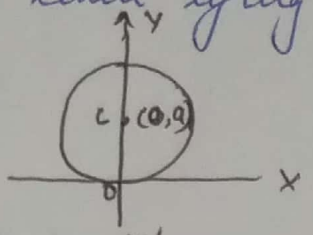
(5) Equation of circle passing through the origin and centre lying on x axis

$$x^2+y^2-2ax=0$$



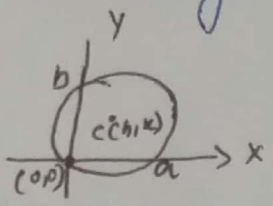
(6) Equation of circle passing through origin and centre lying on Y axis

$$x^2+y^2-2ay=0$$



(7) Equation of the circle through the origin and cutting intercepts a and b on the coordinate axis is

$$x^2+y^2-bx=0$$



Assignment-2

Ques-1 (a) Find the equation of circle with centre $(2,-2)$ and radius 5.

(b) centre $(-\sqrt{2}, -2)$, $r = \sqrt{6}$

Ques 2. Find the centre and radius of circles with following equations

(a) $3x^2 + 3y^2 - 4x + y - 1 = 0$

(b) $(x+2)^2 + (y-1)^2 = 16$

Ques 3 Find the equation of the circles passing through Points $(0,1)$, $(3,-3)$ and $(3,-1)$

(ii) Points $(1,0)$, $(-1,0)$ and $(0,1)$

Ques-4 Find the equation of the circle having

(a) $(-2,5)$ and $(3,4)$ as the end points of its diameter

(b) $(-3,7)$ and $(2,-1)$ as the end points of its diameter also find centre and radius in both cases.

Ques-5 Find the equations of the circle with centre on the line $y = -x$ and has radius 4 and passes through the origin.

Ques-6 Find the equation of the circle with centre $(1,2)$ and which passes through the point $(4,6)$

Ques-7 Find the equation of circle whose centre is $(3,4)$ and which touches the line $5x + 12y = 1$