FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN MAGNETIC FIELD

Let us consider a conductor PQ of length l and area of cross section A. The conductor is placed in a uniform magnetic field of induction B making an angle θ with the field [Fig 3.22].

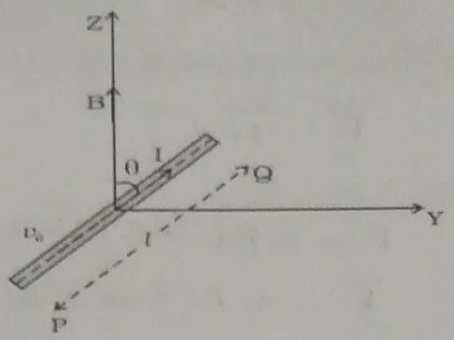

Fig 3.22 Force on a current carrying conductor placed in a magnetic field

A current I flows along PQ. Hence, the electrons are drifted along QP with drift velocity vd. If n is the number of free electrons per unit volume in the conductor, then the current is

$$I = nAv_d e$$

Multiplying both sides by the length l of the conductor,

$$\therefore \; Il = nAv_d el$$

Therefore the current element,

$$\vec{Il} = -nA\vec{v_d}el \quad \text{—— (1)}$$

The negative sign in the equation indicates that the direction of current is opposite to the direction of drift velocity of the electrons.

Since the electrons move under the influence of magnetic field, the magnetic lorentz force on a moving electron.

$$\vec{f} = -e(\vec{v_d} \times \vec{B}) \quad \text{—— (2)}$$

The negative sign indicates that the charge of the electron is negative.

The number of free electrons in the conductor

$$N = nAl \quad \text{——} \quad (3)$$

The magnetic Lorentz force on all the moving free electrons

$$\vec{F} = \vec{N} f$$

Substituting eq (2) and (3) in the above equation

$$\vec{F} = nAl\left(-e\left(\vec{V_d} \times \vec{B}\right)\right)$$

$$\vec{F} = -nAle\,\vec{V_d} \times \vec{B} \quad \text{——} \quad (4)$$

Substituting eq (1) in eq (4)

$$\vec{F} = \vec{Il} \times \vec{B}$$

This total force on all the moving free electrons is the force on the current carrying conductor placed in the magnetic field.

Magnitude of the force

The magnitude of the force is $F = BIl \sin \theta$

(1) If the conductor is placed along the direction of the magnetic field, $\theta = 0^\circ$, Therefore force $F = 0$.

(2) If the conductor is placed perpendicular to the magnetic field, $= 90^\circ$, $F = BIl$ Therefore the conductor experiences maximum force.

Direction of force

The direction of the force on a current carrying conductor placed in a magnetic field is given by Fleming's Left Hand Rule.

The forefinger, the middle finger and the thumb of the left hand are stretched in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, the middle finger points in the direction of the current, then the thumb points in the direction of the force on the conductor.

Force between two Parallel Current Carrying Conductor

AB and CD are two straight very long parallel conductors placed in air at a distance a. They carry currents I1 and I2 respectively.(Fig 3.23) The magnetic induction due to current I1 in AB at a distance a is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \quad \text{——— (1)}$$

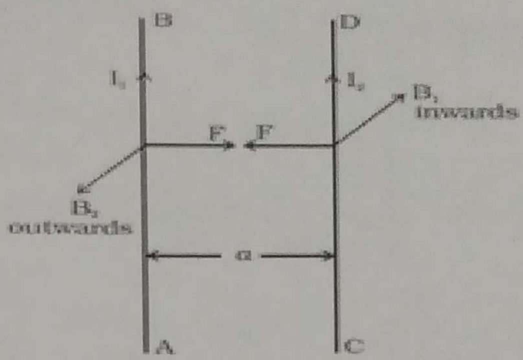

Fig. 3.23 Force between two long parallel current-carrying conductors

This magnetic field acts perpendicular to the plane of the paper and inwards. The conductor CD with current I2 is situated in this magnetic field. Hence, force on a segment of length l of CD due to magnetic field B1 is

$$F = B_1 I_2 l$$

Substituting equation (1)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \quad \text{——— (2)}$$

By Fleming's Left Hand Rule, F acts towards left. Similarly, the magnetic induction due to current I2 flowing in CD at a distance a is

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \quad \text{——— (3)}$$

This magnetic field acts perpendicular to the plane of the paper and outwards. The conductor AB with current I1, is situated in this field. Hence force on a segment of length l of AB due to magnetic field B2 is

$$F = B_2 I_1 l$$

Substituting equation (3) $\therefore F = \frac{\mu_0 I_1 I_2 l}{2\pi a}$ ——— (4)

By Fleming's left hand rule, this force acts towards right. These two forces given in equations (2) and (4) attract each other. Hence, two parallel wires carrying currents in the same direction attract each other and if they carry currents in the opposite direction, repel each other.

Definition of ampere

The force between two parallel wires carrying currents on a segment of length l is

$$F = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

$\therefore$ Force per unit length of the conductor

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

If $I_1 = I_2 = 1A$, $a = 1m$

$$F/l = \frac{\mu_0}{2\pi} \times \frac{1 \times 1}{1} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} Nm^{-1}$$

The above conditions lead the following definition of ampere.

Ampere is defined as that constant current which when flowing through two parallel infinitely long straight conductors of negligible cross section and placed in air or vacuum at a distance of one meter apart, experience a force of 2 × 10-7 Newton per unit length of the conductor.

Torque on a current loop placed in a Magnetic Field.

Let us consider a rectangular loop PQRS of length l and breadth b (Fig). It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction B. Let θ be the angle between the normal to the plane of the loop and the direction of the magnetic field.

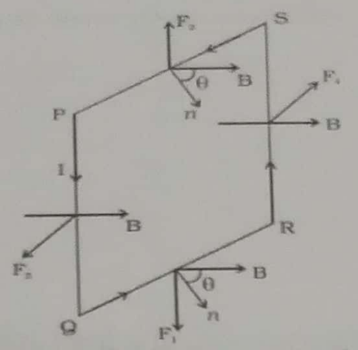

Fig *Torque on a current loop placed in a magnetic field*

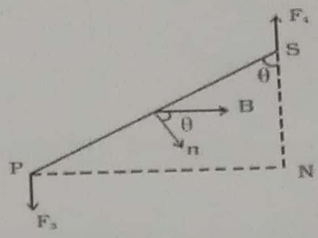

Fig *Torque*

Force on the arm QR. $\vec{F_1} = \overrightarrow{I(QR)} \times \vec{B}$

Since the angle between $\overrightarrow{I(QR)}$ and $\vec{B}$ is $(90^\circ - \theta)$.

Magnitude of the force $F_1 = BIb \sin(90^\circ - \theta)$

ie. $F_1 = BIb \cos\theta$

Force on the arm SP. $\vec{F_2} = \overrightarrow{I(SP)} \times \vec{B}$

Since the angle between $\overrightarrow{I(SP)}$ and $\vec{B}$ is $(90^\circ + \theta)$.

Magnitude of the force $F_2 = BIb \cos\theta$

The force F_1 and F_2 are equal in magnitude, opposite in direction and have the same line of action. Hence their resultant effect on the loop is equal.

Force on the arm PQ. $\vec{F_3} = \overrightarrow{I(PQ)} \times \vec{B}$

Since the angle between $\overrightarrow{I(PQ)}$ and $\vec{B}$ is 90°.

Magnitude of the force $F_3 = BIl \sin 90^\circ = BIl$

F_3 acts perpendicular to the plane of the paper and outwards.

Force on the arm RS. $\vec{F_4} = \overrightarrow{I(RS)} \times \vec{B}$

Since the angle between $\overrightarrow{I(RS)}$ and $\vec{B}$ is 90°.

Magnitude of the force $F_4 = BIl \sin 90^\circ = BIl$

Magnitude of the force F4 = $BIl \sin 90° = BIl$

F4 acts perpendicular to the plane of the paper and inwards. The forces F3 and F4 are equal in magnitude, opposite in direction and have different lines of action. So, they constitute a couple.

Hence, Torque = $BIl \times PN = BIl \times PS \times \sin \theta$

$= BIl \times b \sin \theta = BIA \sin \theta$

If the coil contains n turns, $\tau = nBIA \sin \theta$

So, the torque is maximum when the coil is parallel to the magnetic field and zero when the coil is perpendicular to the magnetic field.