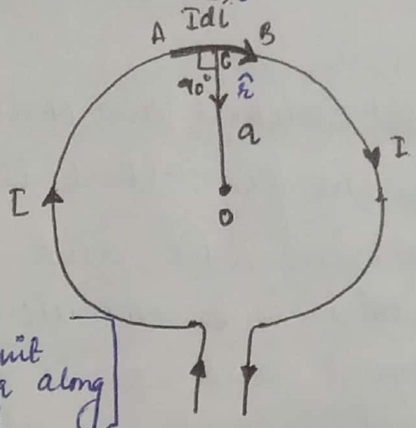
Magnetic field at the centre of a circular loop.

Consider a circular loop of radius a, centre O and carrying current I. To find magnetic field at the centre of the loop (at O), consider an infinitesimally small length $AB = dl$ of the loop. Join the centre O of the loop to the middle pt. C of the current element AB. ∴ Clearly $\angle ACO = 90^\circ$.



Now according to Biot Savart's law, the magnetic field at point O due to the current element AB is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\vec{dl} \times \hat{r}}{a^2}$$

[$\hat{r} \rightarrow$ unit vector along CO]

where $I\vec{dl} = I(\vec{AB})$ is a vector in the direction of flow of current through AB and $\hat{r}$ is a unit vector along CO.

The magnetic field $d\vec{B}$ at point O due to the small current element AB points in the direction of $I\vec{dl} \times \hat{r}$. i.e perpendicular to the plane of the loop and in inward direction.

Since angle b/w $I\vec{dl}$ & $\vec{r}$ is 90°, the magnitude of magnetic field at pt. O due to current element AB is

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{a^2}$$

∴ Magnetic field due to whole circular loop ⇒

$$B = \oint dB = \oint \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{a^2} = \frac{\mu_0}{4\pi} \cdot \frac{I}{a^2} \oint dl$$

Now $\oint dl = 2\pi a \rightarrow$ circumference of loop

$$\therefore \quad B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a^2} \times (2\pi a)$$

or

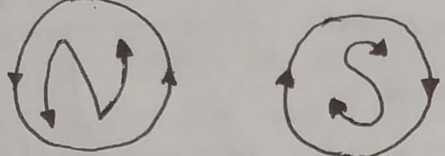
$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a}}$$

In case the circular loop has 'n' turns, then

$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a}}$$

Direction of Magnetic Field :- Magnetic field B due to whole circular loop will be perpendicular to the plane of the loop and in inward direction. In general, if current through the loop flows in clockwise direction, then direction of Magnetic field at its centre is perpendicular to the plane of the loop and in inward direction.

On the other hand, if the current flows in anti-clockwise direction, then direction of magnetic field at its centre is perpendicular to the plane of the loop and in outward direction.



⇒ Face of the loop, in which the current appears to flow anticlockwise → acts as magnetic north pole and if current appears to flow clockwise then the face acts as magnetic south pole.

In both cases, the magnetic field at the centre of the loop is directed along normal to the plane of loop.

Ques The plane of a circular coil is horizontal. It has 10 turns, each of 8 cm radius. A current of 2A flows through it, which appears clockwise from a point vertically above it. Find the magnitude and direction of the magnetic field at the centre of coil due to the current.

Ampere's Circuital Law :-

It states that the line integral of magnetic field ($\vec{B}$) around any closed path or circuit is equal to μ_0 (absolute permittivity of free space) times the total current (I) threading the closed circuit. Mathematically,

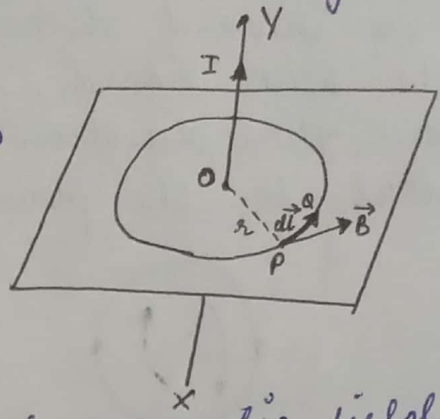
$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I} \ast$$

Ampere Circuital Law Contd...

To Prove Ampere's circuital law for a circular Path :-

Consider a long straight conductor XY, such that a current I flows through it from X to Y. Due to flow of current through the conductor, magnetic field will be produced around it.

Consider a circular path of radius r, such that conductor XY passes through its centre O. Let $\vec{PQ}\ (=\vec{dl})$ be a small element of the circular path, and $\vec{B}$ be the strength of the magnetic field at point P.



According to right hand thumb rule, the direction of magnetic field at point P is along tangent to the circular path at that point. Obviously, the vectors $\vec{B}$ and $\vec{dl}$ are along the same direction. Hence, the line integral of magnetic field over the circular closed path is given by

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos 0^\circ = \oint B\,dl \quad \text{--- (1)}$$

The magnitude of magnetic field due to current carrying straight conductor at point P is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r} \quad \text{--- (2)}$$

Putting the value of B from (2) to (1)

$$\oint \vec{B}\cdot\vec{dl} = \oint \frac{\mu_0}{4\pi}\,\frac{2I}{r}\,dl = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}\oint dl$$

Now $\oint dl = 2\pi r$ (circumference of circular Path)

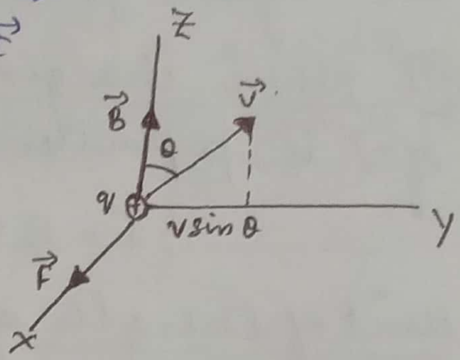
$$\oint \vec{B}\cdot\vec{dl} = \frac{\mu_0}{4\pi}\cdot\frac{2I}{\cancel{r}} \times \cancel{2\pi}\cancel{r}$$

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I}$$

Force on a charge moving in a Magnetic Field :-

Consider that a charge $+q$ is moving inside the magnetic field of strength $\vec{B}$. Suppose that magnetic field acts along Z axis, while the charge $+q$ moves with velocity $\vec{v}$ in YZ plane making an angle θ with the direction of $\vec{B}$

As such, the charge $+q$ experiences force $\vec{F}$ along x axis. The magnitude of force is found to be :-



(I) $F \propto B$ [Directly Proportional to strength of Mag. Field strength]

(II) $F \propto q$

(III) $F \propto v \sin\theta$.

combining these three, we have

$$F \propto Bqv \sin\theta$$

or $F = K\, Bqv \sin\theta$. $\quad K \rightarrow$ const. of Proportionality.

The value of K is found to be 1 when other quantities measured in S.I units.

$$\boxed{F = Bqv \sin\theta}$$

The force on a charged particle moving inside a magnetic field is called Magnetic Lorentz Force.

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(II) When charged particle moves parallel to the magnetic field

If $\vec{v}$ is $\parallel$ to $\vec{B}$ then $\theta = 0°$ or $180°$ then

$$F = Bqv(0) = 0 \qquad \{\sin\theta = 0 \{\theta = 0°, \theta = 180°\}\}$$

No magnetic force acts on the charge when it moves parallel to the magnetic field.

(III) When charged particle moves $\perp$ar (perpendicular) to the magnetic field:

If $\vec{v}$ is perpendicular to $\vec{B}$ then $\theta = 90°$. In that case.

$$F = Bqv\sin\theta \Rightarrow F = Bqv \,(\text{max}) \qquad \{\sin 90° = 1\}$$

Unit of strength of magnetic field :-

In S.I, the unit of strength of magnetic field is Tesla [T].

Ques. A charge of 3C is moving with velocity $\vec{v} = (4\hat{i} + 3\hat{j})\,ms^{-1}$ in a magnetic field $\vec{B} = (4\hat{i} + 3\hat{j})\,Wb\,m^{-2}$. Find the force acting on the test charge.

Lorentz Force :- When a charge q moves with velocity $\vec{v}$ inside magnetic field of strength $\vec{B}$, then force on the charge

$$\vec{F} = q(\vec{v} \times \vec{B})$$

It is called magnetic Lorentz Force on the charge due to its motion inside magnetic field.

If in addition to magnetic field, an electric field of strength $\vec{E}$ also acts, then total force on charge q is given by

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

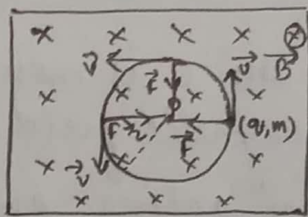
When a charged particle having charge q moves inside a magnetic field $\vec{B}$ with velocity $\vec{v}$, it experiences a force given by

$$\vec{F}_m = q(\vec{V} \times \vec{B}) \quad \text{— (1)}$$

The direction of the force on the charged particle due to magnetic field is always in a direction ⊥ar to both $\vec{v}$ and $\vec{B}$.

a) when $\vec{V}$ & $\vec{B}$ are perpendicular :-

In this case $\theta = 90°$ so that $F = qvB\sin 90° = qvB$.
This is the maximum force of constant magnitude and this force always acts perpendicular to the direction of motion of charge so the charge moves along circular path and required centripetal force is provided by the magnetic force.



centripetal force on moving charge be given as

$$F_{cp} = \frac{mv^2}{r} \longrightarrow (2)$$

where 'm' is the mass of the charge and 'r' be the radius of circular path. From (1) & (2)

$$\vec{F}_m = F_{cp}$$

$$\boxed{qvB = \frac{mv^2}{r}}$$

or

$$\boxed{r = \frac{mv}{qB}} \quad \text{- (3)}$$

It follows that :-

(1) Faster the particle moves (v large), the larger is the radius of circular path.

(2) Heavier the charged particle (m large), the larger is radius

(3) greater $\vec{B}$ means smaller the radius of circular path.

The period of circular motion of a charged particle is given by T and angular frequency ω. Now we know that for uniform circular motion

$$V = r\omega \quad \text{or} \quad \omega = \frac{V}{r}$$

Putting the value of v from ③

$$\boxed{\omega = \frac{Bq}{m}} \quad \text{and } T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = \frac{2\pi m}{Bq}}$$

From above two expressions for ω & T, it follows that period of circular motion and angular frequency of a charged particle moving in uniform magnetic field neither depends upon magnitude of its velocity nor upon the radius of the circular path.

Ques An e^- is moving with a velocity 1.2×10^7 m/sec perpendicular to a magnetic field of 5×10^{-4} Wb/m^2 strength. Calculate-

(I) Force acting on the electron

(II) Radius of its path

Given charge on $e^- = 1.6 \times 10^{-19}$ C, Mass of $e^- = 9.1 \times 10^{-31}$ kg

Ques Find out the magnetic field due to a solenoid using ampere's circuital law.